Decentralized formation control considering information propagation

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Abstract

There is a growing interest in formation flight of satellites and its control strategy. The centralized control architecture is optimal when all information is shared by every member in the formation, however, it possibly shows unstable behavior in a realistic situation where the information exchange cost (for example, communication load) is large and the information can be only partially shared. This paper proposes a new decentralized control strategy for multi-vehicle formation control, which effectively propagates the information between adjacent agents and uses the information in the control calculation. The strategy can overcome the drawback of the centralized approach while maintaining the communication load very low. The effectiveness of the proposed strategy is numerically verified by applying to one-dimensional car train control problem and two-dimensional in-plane satellite formation control problem.

1. Introduction

Recently the formation flying has become very common and its keeping strategies are now of great interest in Astrodynamics these days. It may be simply true that the optimal station keeping is realized only when every spacecraft information is shared by every member in the formation. This is the case called “full informed” state and the strategy taken results in a centralized control. However, in case the number of members constituting a formation becomes infinitely large, the communication load of the system becomes divergent and the states uncertainty may deform the relative geometry before the entire information is exchanged.

Instead of this centralized control law, it is practical to apply a localized control law that requests only the information of relative motion among a few spacecraft adjacent to each other. In this case, the strategy will be a decentralized one. In the decentralized control, as the information is not shared by every member, the optimal control is impossible and non-optimal transition behavior is observed. A typical example of this non-optimal transition behavior is the traffic jam phenomenon in highways on the ground. When the head car of platoon rises/slowdowns in the speed, oscillatory phenomenon of the car density like compressional wave is observed, and in the worst case, the platoon shows divergent behavior. This is because that the member cannot use the full information of the platoon or formation. The partially informed control makes the transition behavior not uniformly converged to the goal and the settle time increases. This problem is also observed in the satellite formation control and some appropriate control strategy should be implemented.

This paper first deals with the formation behavior via z-transformation method. The z-transformation expression can easily express the transition behavior of the partially informed formation. A uniformly convergent strategy is proposed in the z-transformed formulation, which physically indicates that the information of the relative motion and the resulted control input is propagated from one agent to the adjacent one. This information propagation scheme was proved to stabilize the formation behavior.

2. One-dimensional platoon control

This section deals with the control of a car platoon as a typical example of one-dimensional formation control.

2.1 Mathematical formulation via z-transform

Fig. 1 shows the typical one-dimensional formation. A specific position of each vehicle is denoted as xi and an acceleration input to the each vehicle is denoted as ui. That is:

$$\ddot{x}_i = u_i \tag{1}$$

where, using y_i as a distance between vehicles, if the definition is given as follows:

$$y_i \equiv x_{i-1} - x_i \tag{2}$$

and thus

$$\ddot{y}_i = u_{i-1} - u_i \tag{3}$$

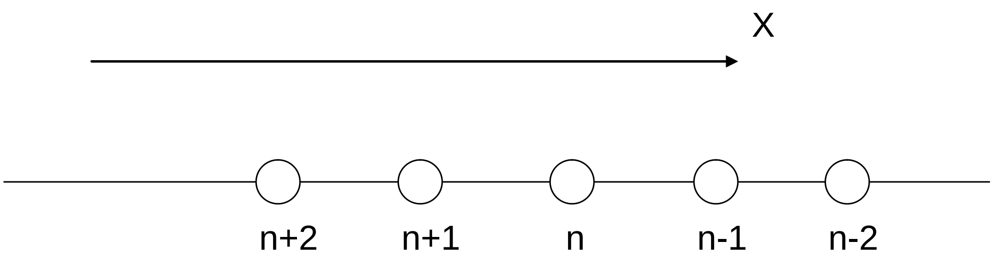

Fig. 1: One-dimensional formation

In usual traffic, the driver usually inputs the control of $u_i = \omega^2 (y_i - \bar{y})$ to maintain the inter-vehicle distance, which results in

$$\ddot{y}_i + \omega^2 y_i = \omega^2 y_{i-1} \tag{4}$$

This relation very simply indicates how the traffic flow is destabilized. The foregoing cars naturally have same characteristic frequency and the following cars must suffer from the oscillatory reference input, to which the acceleration resonances. Since most of drivers have very similar characteristic frequency, this resonant tendency governs and make the cars flow density amplitude increased seriously.

This is further z-transformed spatially to give a result expressed as follows:

$$\ddot{y}_z = \frac{1-z}{z} u_z \tag{5}$$

The z-operator included in the right-hand side indicates a delayed transmission, and it is essential to offset and delete the delayed transmission from the viewpoint of control logic in order to provide a stable vehicle train control. Although this has been also previously reported, a novel interpretation thereof will be herein added. A first consideration is directed to the case where a driver attempts to control a distance between his/her vehicle and a vehicle traveling ahead thereof. In this case, assuming that a speed error feedback gain and an inter-vehicle distance error feedback gain are denoted as Kd and Kp, respectively, then the control logic is expressed to be:

$$u_i = -K_d(\dot{x}_i - \bar{\dot{x}}_i) + K_p(y_i - \bar{y}_i) \tag{6}$$

or, when z-transformed,

$$u_z = -K_d(\dot{x}_z - \bar{\dot{x}}_z) + K_p(y_z - \bar{y}_z) \tag{7}$$

It is to be noticed that in this expression, a variable added with a lateral bar on top indicates a target value for the variable. Since from the following expressions:

$$y_z = \frac{1-z}{z} x_z \tag{8}$$

$$\dot{x}_z = \frac{z}{1-z} \dot{y}_z \tag{9}$$

are given (attention should be pain on the minus sign), a closed-loop characteristic follows

$$\ddot{y}_z = -K_d \left\{ \dot{y}_z - \frac{1-z}{z} \bar{\dot{x}}_z \right\} - K_p \frac{z-1}{z} \{ y_z - \bar{y}_z \} \tag{10}$$

Herein, a simple interpretation of the above characteristic is given. Without loosing generality, $\bar{y}_z = 0$ is assumed, where, Kd is sufficiently small and replaced as $K_p = \Omega^2$, then a solution under an initial condition of yzo is given in

$$y_z \cong \{ y_{z0} - \bar{y}_z \} \cos\left(\sqrt{1 - \left(\frac{1}{z} \right)} \Omega t \right) + \bar{y}_z \tag{11}$$

If the above equation is expanded by (1/z) and resultant coefficients are evaluated, then the inter-vehicle distance can be determined. A first term gives a propagation of a difference between an initial condition and a target of control to a vehicle train, wherein the n-th inter-vehicle distance from the head of the train can be given by a expansion factor of (1/z)n , and in this simplified model, it may be given in

$$\frac{1}{n!}\left(\frac{1}{2}\Omega^2 t^2\right)^n \cos\left(\Omega t - \frac{n\pi}{2}\right) \tag{12}$$

Theoretically, the phase should be different by 90 degrees for every inter-vehicle distance. This amplitude may be:

$$a_n = \frac{1}{n!}\left(\frac{1}{2}\Omega^2 t^2\right)^n \tag{13}$$

And if within an finite time, $a_{n+1}/a_n \to 0$, thus, the inter-vehicle distance would not diverge. Contrary, for a finite train length, the inter-vehicle distance would diverge over time. In a sequence of inter-vehicle distance, ultimately, the amplitude causes a vibration expressed as:

$$|y_{z0} - \bar{y}_z|\left(\frac{1}{2}\Omega^2 t^2\right) \tag{14}$$

to be produced in the automobiles in the downstream train, which is further superposed, leading to the traffic congestion. Fig. 2 shows the example of such divergent behavior.

It could be said that in order to avoid the congestion, effectively $\Omega \ll 1$ should be given to reduce sensitivity for triggering the acceleration or deceleration operation in response to the inter-vehicle distance, though it could not be essentially avoided that the vibration propagated divergently to the automobiles in the downstream train would be induced. Consequently, if any disturbance in the inter-vehicle distance occurs, then the congestion should be induced without exception in the conventional method, in which the control is given by the driver who judges a distance with respect to the automobile ahead.

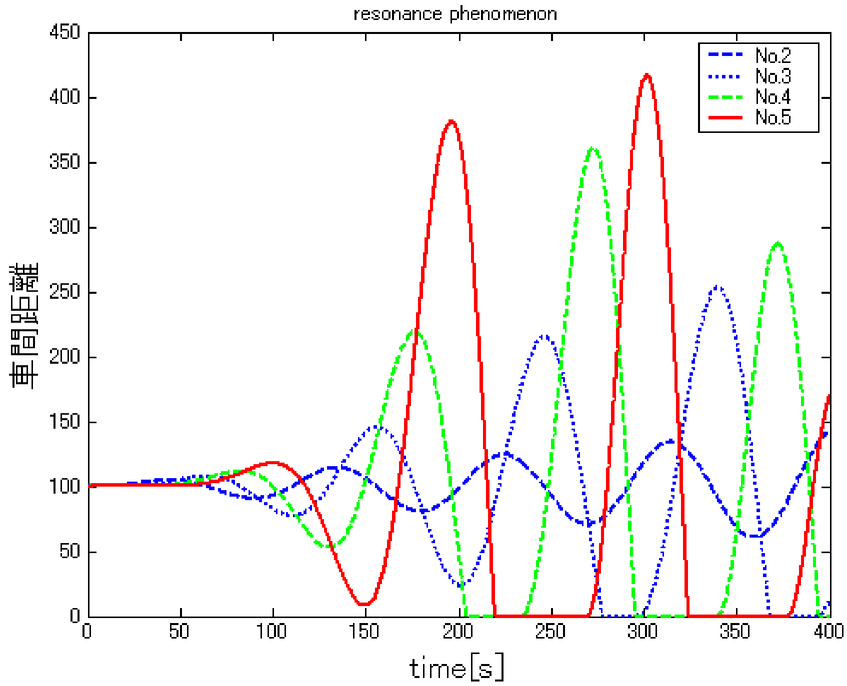

Fig. 2: Divergent behavior of car train

2.2 Extension to a continuous system

To understand the dynamics of the system more deeply, the differential equation for discrete integer i (i=0,1,…) is extended to any continuous i between 0 and 1. By eq.(3) and (6), the discrete expression is

$$\ddot{y}_i = -K_d \dot{y}_i - K_p (y_i - y_{i-1}) \tag{15}$$

where, it is assumed that $\bar{y}_i$ and $\bar{\dot{y}}_i$ are same for all i. If the variable i takes a continuous value between 0 and 1, this equation can be rewritten as

$$\begin{aligned} \ddot{y}_i &= -K_p (y_i - y_{i-1}) - K_d \dot{y}_i \\ &= -K_p \frac{y_i - y_{i-1}}{\frac{i}{n} - \frac{i-1}{n}} \frac{1}{n} - K_d \dot{y}_i \\ &= -K_p \frac{\partial y_i}{\partial(\frac{i}{n})} \frac{1}{n} - K_d \dot{y}_i \end{aligned} \tag{16}$$

So, the continuous expression can be obtained as

$$\ddot{y}_i = -K_p \frac{\partial y_i}{\partial i} - K_d \dot{y}_i \tag{17}$$

If the general solution of eq. (17) is assumed to be

$$y_i = \alpha(t)\beta(i) \tag{18}$$

the following differential equation can be obtained by eq. (17) and (18):

$$\frac{\ddot{\alpha}}{\alpha} + K_d \frac{\dot{\alpha}}{\alpha} = -K_p \frac{\beta'}{\beta} = \lambda \quad (\text{const}) \tag{19}$$

Therefore, by introducing a constant □, the following two governing equations are obtained:

$$\ddot{\alpha} + K_d \dot{\alpha} - \lambda \alpha = 0 \tag{20}$$

$$\beta' + \frac{\lambda}{K_p} \beta = 0 \tag{21}$$

Clearly, when □ > 0, □ is unstable and will diverge. As □ is determined by initial condition, eq.(20) states that the time history of the inter-vehicle distance can be unstable depending on the initial condition. Eq.(21) shows that the inter-vehicle distance can be propagated along the platoon. This phenomenon is interpreted as the traffic jam.

2.3 Control law with information propagation structure

In this section, a control law is proposed which has an information propagation scheme between adjacent vehicles.

By eq.(7) and (8), the conventional control law is rewritten as

$$u_z = -K_d(\dot{x}_z - \dot{\bar{x}}_z) + \frac{1-z}{z} K_p (x_z - \bar{x}_z) \tag{22}$$

Here, we add a new term into the above control law to obtain a new control law as

$$u_z = -K_d(\dot{x}_z - \dot{\bar{x}}_z) + \frac{1-z}{z} K_p (x_z - \bar{x}_z) - \frac{1}{z} K_p (x_z - \bar{x}_z) \tag{23}$$

We define the additional term $-\frac{1}{z} K_p (x_z - \bar{x}_z)$ as w_i. w_i can be interpreted as follows:

$$-\frac{1}{z} K_p (x_z - \bar{x}_z) = -K_p (x_{i-1} - \bar{x}_{i-1}) = K_p (y_{i-1} - \bar{y}_{i-1}) - K_p (x_{i-2} - \bar{x}_{i-2}) \,□\, w_i \tag{24}$$

Then, the control law can be divided into these two equations

$$u_i = -K_d(\dot{x}_i - \dot{\bar{x}}_i) + K_p (y_i - \bar{y}_i) + w_i \tag{25}$$

$$w_i = w_{i-1} + K_p (y_{i-1} - \bar{y}_{i-1}) \tag{26}$$

where, w_i can be considered as information propagated from foregoing car. And the dynamics of the system is given by

$$\ddot{y}_z = -K_d \left\{ \dot{y}_z - \dot{\bar{y}}_z \right\} - K_p \{ y_z - \bar{y}_z \} \tag{27}$$

or

$$\ddot{y}_i = -K_d \left\{ \dot{y}_i - \dot{\bar{y}}_i \right\} - K_p \{ y_i - \bar{y}_i \} \tag{28}$$

It can be said that by accumulating and propagating the information wi, the dynamics which was affected by the foregoing car (see eq.(4)) becomes independent of i, and the behavior of the inter-vehicle distance can be stable.

By extending eq.(28) to a continuous expression, we can obtain

$$\ddot{y}_i = -K_p(y_i - \bar{y}_i) - K_d \dot{y}_i \tag{29}$$

which does not contain the partial derivative with respect to i. This means that the dynamics is independent in the along-track direction. The time response can be stable by appropriately selecting K_p and K_d.

2.4 Effect of information propagation delay

The previous section assumed that there is no information propagation delay in the system. This section introduces the effect of information propagation delay into the system dynamics and investigates the resulting dynamics.

Instead of the propagation scheme eq.(26), the following scheme is assumed:

$$w_i = w_{i-1}(t-\Delta t) + K_p(y_{i-1}(t) - \bar{y}_{i-1}) \tag{30}$$

where Δt is the delay time between adjacent vehicles. By accumulating this equation,

$$w_i = \sum_{k=0}^{i-1} K_p(y_k(t-k\Delta t) - \bar{y}_k) \tag{31}$$

So, the control law (eq.(25)) becomes

$$u_i = -K_d(\dot{x}_i(t) - \bar{\dot{x}}_i) + K_p(y_i(t) - \bar{y}_i) + \sum_{k=0}^{i-1} K_p(y_k(t-k\Delta t) - \bar{y}_k) \tag{32}$$

By substituting this equation into eq.(3), we can obtain the Laplace-transformed expression as

$$s^2 Y_i = K_p \frac{z-1}{z}(Y_i - \bar{y}_i) - K_d s Y_i + K_p \frac{1}{z} e^{-\Delta ts}(Y_i - \bar{y}_i) \tag{33}$$

Then, by using $Y_{i-1} \approx Y_i - Y_i'$, $Y_i - Y_{i-1} = \frac{\partial Y_i}{\partial i} \quad (n \to \infty)$, continuous expression is obtained as:

$$[s^2 + K_d s + K_p e^{-\Delta ts}](Y_i - \bar{y}_i) + K_p(1 - e^{-\Delta ts})\frac{\partial Y_i}{\partial i} = 0 \tag{34}$$

The inverse Laplace transform of this equation leads to the following time-domain expression:

$$\ddot{y}_i(t) + K_d \dot{y}_i(t) + K_p(y_i(t-\Delta t) - \bar{y}_i) + K_p \frac{\partial \dot{y}_i(t)}{\partial i} - K_p \frac{\partial \dot{y}_i(t-\Delta t)}{\partial i} = 0 \tag{35}$$

Here, we assume that Δt is small and $y_i(t-\Delta t) \approx y_i(t) - \dot{y}_i(t)\Delta t$,

$$\ddot{y}_i(t) + (K_d - \Delta t K_p)\dot{y}_i(t) + K_p(y_i(t) - \bar{y}_i) + \Delta t K_p \frac{\partial \dot{y}_i(t)}{\partial i} = 0 \tag{36}$$

is obtained.

When the following general solution is assumed and substituted into eq.(36),

$$\dot{y}_i = \alpha(t)\beta(i) \tag{37}$$

we can obtain the equation of α and β as

$$\frac{\dot{\alpha}}{\alpha} + (K_d - \Delta t K_p) + K_p \frac{\int \alpha dt}{\alpha} = -\Delta t K_p \frac{\beta'}{\beta} = \lambda \quad \text{(const)} \tag{38}$$

Therefore, by introducing a constant λ, the following two governing equations are obtained:

$$\ddot{A} + (K_d - \Delta t K_p - \lambda)\dot{A} + K_p A = 0 \tag{39}$$

$$\beta' = -\frac{\lambda}{\Delta t K_p}\beta \tag{40}$$

where $A = \int \alpha dt$. It should be noted that $\lambda \to \infty$ if $\Delta t \to 0$ (see eq.(38)). The general solution of eq.(36) is written as follows:

$$y_i(t) = A(t)\beta(i) + \bar{y}_i \tag{41}$$

By comparing eq.(39) and (29), the propagation delay □t can make the system unstable and the traffic jam can occur. By also comparing eq.(40) and eq.(21), the tendency of how the system becomes unstable can be controlled by the value □t. That is, the new control law with information propagation structure can suppress the traffic jam by selecting appropriate □t, while simple PD control (conventional control) can never.

2.5 Numerical simulation results

<u>Simulation results as discrete system</u>

The performance of the conventional control law and the new one is evaluated by numerical simulations. The system is assumed to be discrete system (section 2.1).

Fig. 3 and 4 show the traffic control example. Distance between 1st and 2nd cars abruptly extends to 160m and the distance between 5th and 6th cars suddenly becomes 150m. Here is assumed the driving skill is common. Fig. 4 shows the result obtained by the usual control law, eq. (13). Rear cars' distances diverge gradually. Fig. 5 shows the result obtained by the new control law shown in eq. (21). As this figure shows, the Dramatic improvement is obtained. Small localized oscillation is found but is damped quickly.

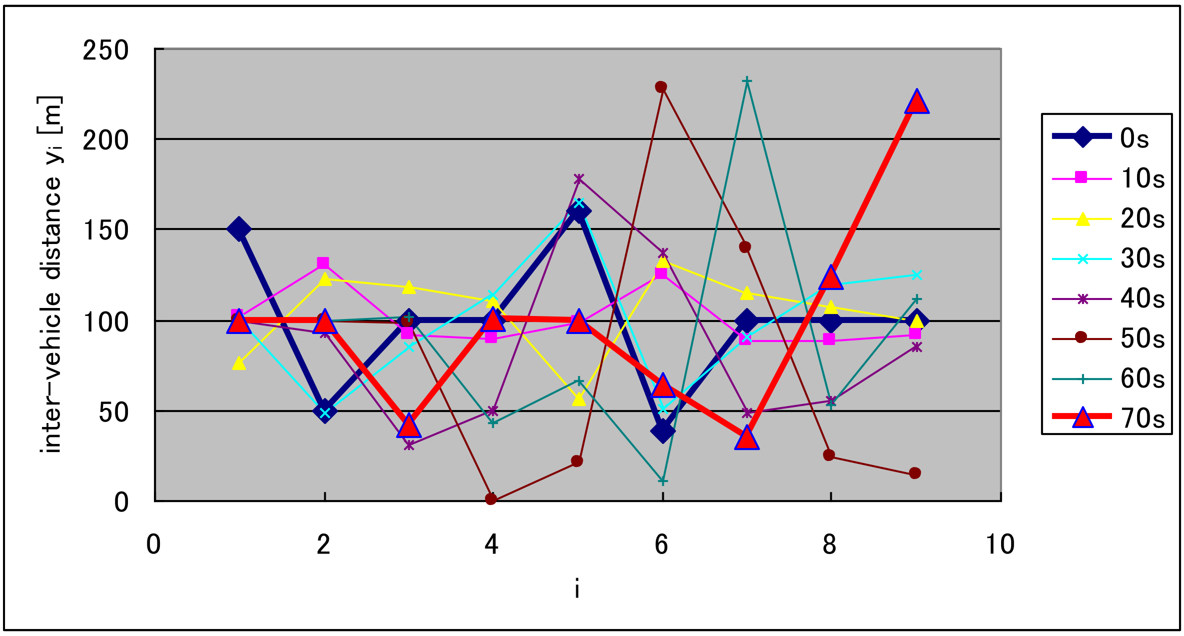

Fig. 3: Control result by conventional control law

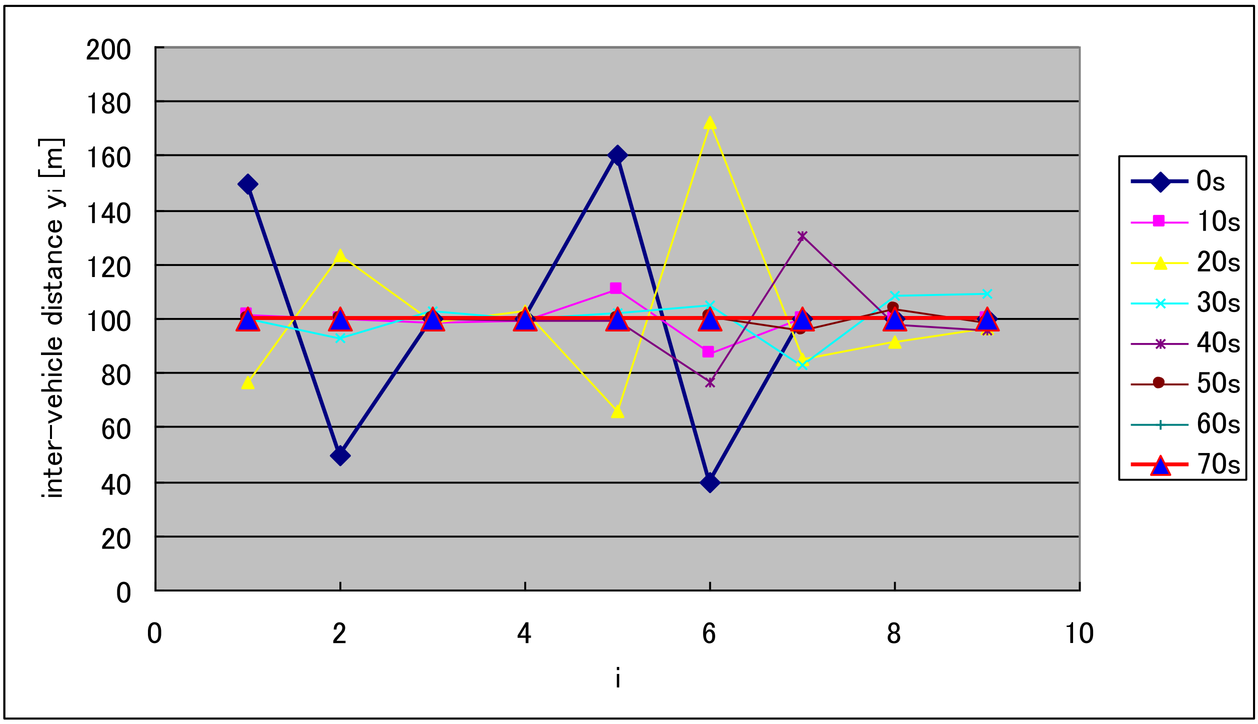

Fig. 4: Control result by new control law

Simulation results as continuous system

Next, simulations are performed as a continuous system, and the effect of information propagation which is discussed in section 2.4 is evaluated.

Fig. 5 and 6 shows the comparison of traffic control for a continuous system, where no information propagation delay is applied. The feedback gain Kp and Kd was 0.5/100, 1.0 [m/s2] respectively. The initial disturbance condition was such that the inter-vehicle distance 0.99 was applied at i=0.5. The horizontal axis in the figure shows continuous i, and the vertical axis is the transient time [s]. In Fig. 5, divergent behavior is observed from 20 sec and the behavior propagates to i=0.4-0.7 at 60 sec, so the conventional control could not suppress the divergence. On the other hand, in Fig. 6, the new control law with information propagation structure could suppress the initial disturbance within 10 seconds.

Then, the effect of the information propagation delay is imposed. Fig. 7 and Fig. 8 shows the simulation results of new control law when the propagation delay is 1sec and 2sec respectively. In Fig. 7, the initial disturbance is suppressed by 60 seconds. However, in Fig. 8, the initial disturbance is growing. So, the new control law cannot overcome the time delay of 2 seconds.

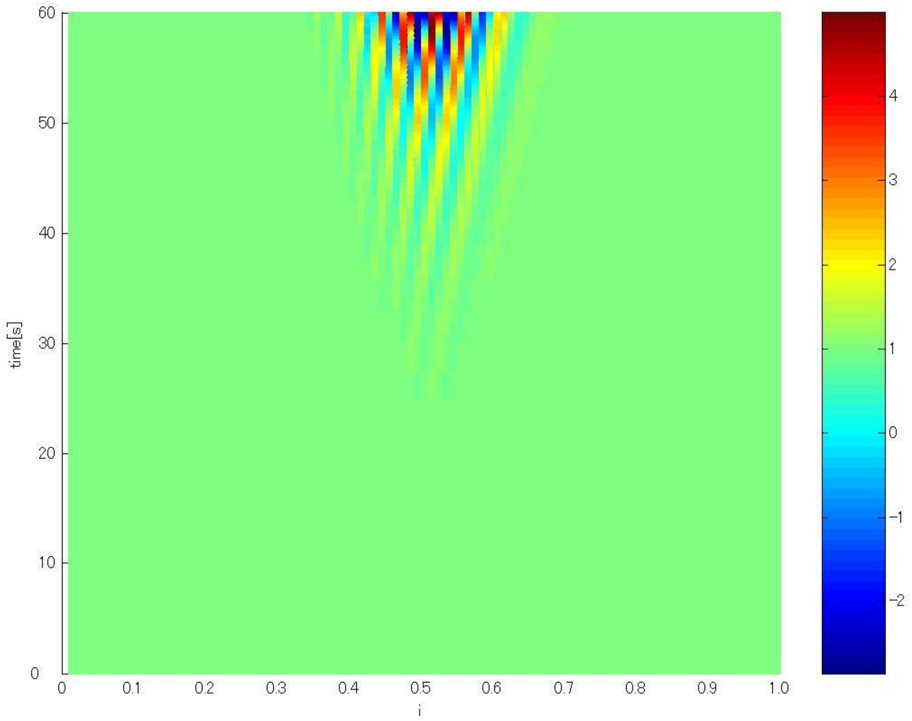

Fig. 5: Control results of conventional control law (continuous system)

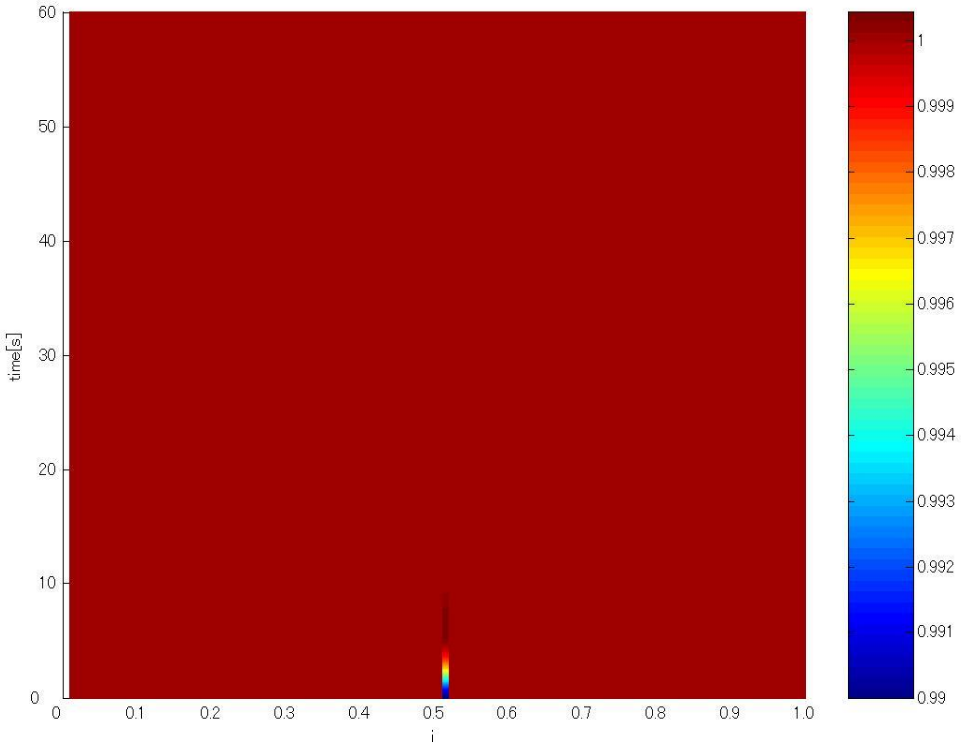

Fig. 6: Control results of new control law (continuous system)

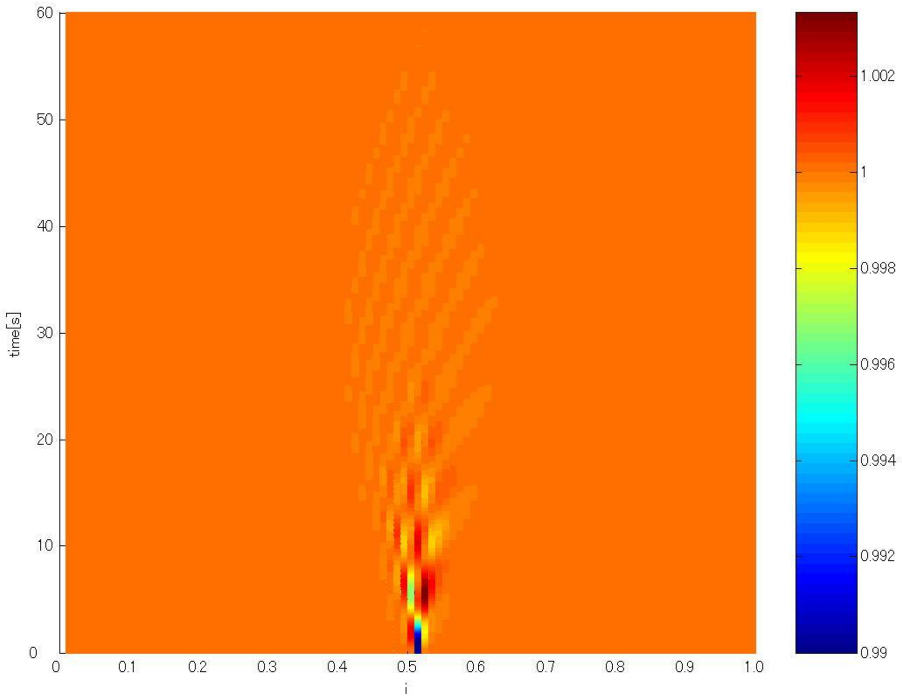

Fig. 7: Control results of new control law (Δt=1 [s])

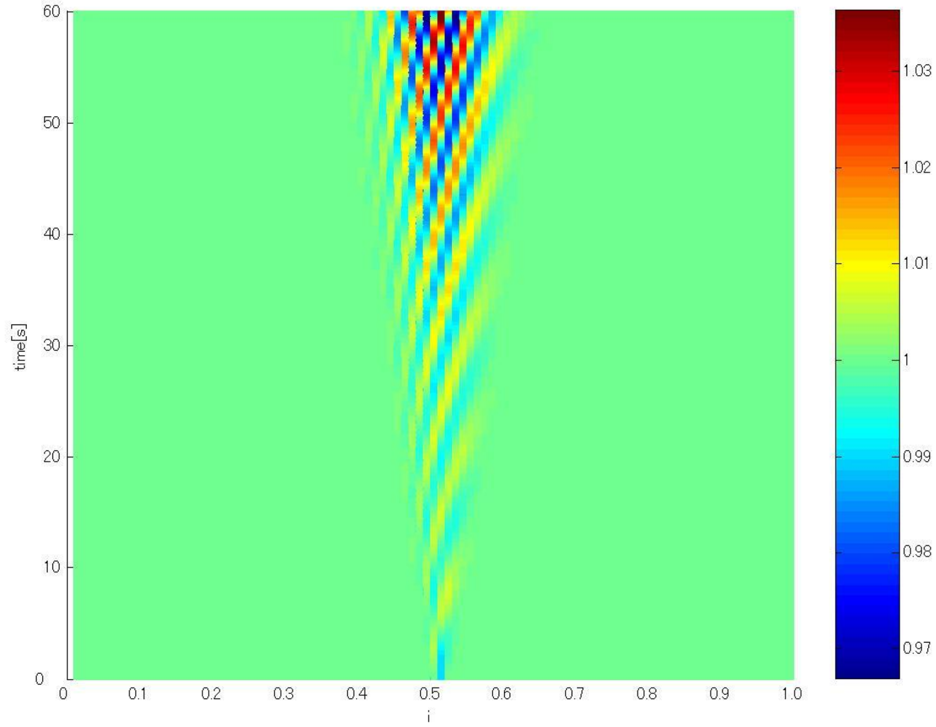

Fig. 8: Control results of new control law (Δt=2 [s])

3. Application to satellite formation control

In this section, the proposed control law with information propagation scheme is applied to satellite formation control problem.

3.1 Mathematical formulation

Here, we consider two-dimensional (in-plane) formation keeping problem around a circular reference orbit.

The equation of relative motion is described by the two-dimensional Hill's equation:

$$\begin{aligned} \ddot{x} - 2n\dot{y} - 3n^2 x = u \\ \ddot{y} + 2n\dot{x} = v \end{aligned} \tag{42}$$

where n denotes to the orbital angular velocity of the reference circular trajectory and x and y is the relative position of a satellite in the anti-nadir and along-track direction respectively.

When a contraction of $x' = x,\ y' = y/2$ is used, and the rotation coordinate transformation of

$$p = \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \tag{43}$$

is introduced, the equations of motion is altered to the following time-varying expression.

$$\ddot{p} + A(\theta)\dot{p} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ \frac{1}{2}v \end{pmatrix} = \hat{U} \tag{44}$$

where

$$A(\theta) = n \begin{pmatrix} 3\sin\theta\cos\theta & 1-3\cos^2\theta \\ -1+3\sin^2\theta & -3\sin\theta\cos\theta \end{pmatrix} \tag{45}$$

and θ is the true anomaly of the reference trajectory. The p-coordinate is rotating in reverse direction of the reference orbit as shown in Fig. 9.

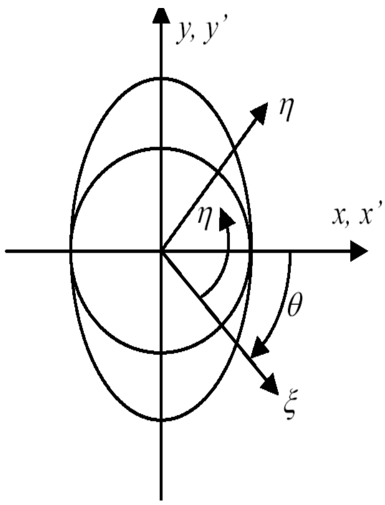

Fig. 9: Coordinate transformation

In this coordinate, it is clear that a trivial solution, a constant vector always satisfies the equation above. This corresponds to the state in which the formation is frozen along similar ellipse trajectories, i.e. like cart wheel orbits. This section deals with a station-keeping problem in this coordinate.

The relative motion in the coordinate can be expressed as:

$$\Delta\ddot{p}_{ij} + A\dot{p}_{ij} = \hat{U}_{i-1,j-1} - \hat{U}_{ij} = \begin{pmatrix} \hat{u}_{i-1} - \hat{u}_i \\ \hat{v}_{j-1} - \hat{v}_j \end{pmatrix} \tag{46}$$

where $\Delta p_{ij} = p_{i-1,j-1} - p_{ij}$. This can be converted via z-transformation as:

$$\Delta\ddot{p}_z + A\dot{p}_z = \begin{pmatrix} \frac{1}{z_\xi} - 1 & 0 \\ 0 & \frac{1}{z_\eta} - 1 \end{pmatrix} \hat{U}_z \tag{47}$$

By applying the new control law proposed in the previous section,

$$\hat{U}_z = \begin{pmatrix} \frac{z_\xi}{z_\xi - 1} & 0 \\ 0 & \frac{z_\eta}{z_\eta - 1} \end{pmatrix} (c\dot{p} + kp) \tag{48}$$

This is decomposed by the inverse z-transform as:

$$\begin{aligned} u &= \cos\theta\alpha_n + \sin\theta\beta_m \\ \frac{1}{2}v &= -\sin\theta\alpha_n + \cos\theta\beta_m \\ \alpha_n &= \alpha_{n-1} + c\Delta\dot{\xi}_n + k\Delta\xi_n \\ \beta_m &= \beta_{m-1} + c\Delta\dot{\eta}_m + k\eta_m \end{aligned} \tag{49}$$

where α_n and β_m are the value to be propagated between adjacent satellites.

3.2 Numerical simulation results

Fig. 10 shows the trajectories of four satellites that are initially positioned at the origin (reference trajectory). The target position of each satellite in the □□□ coordinate is (1,1), (1,2), (2,2) and (4,2) respectively. The new control scheme guarantees uniform convergence to the target point and the entire formation is controlled.

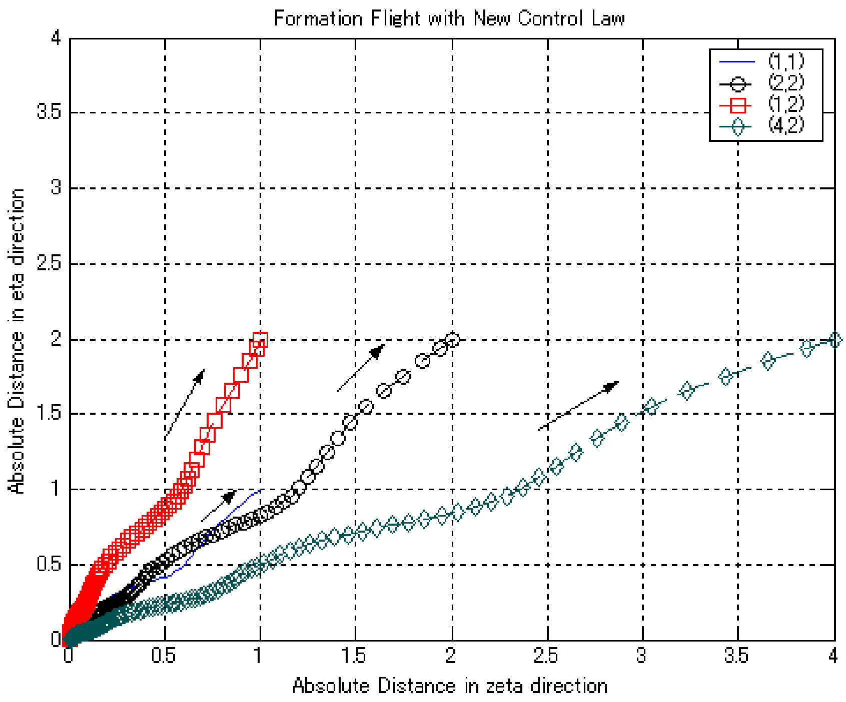

Fig. 10: Results of 2-dimensional formation control

4. Conclusion

This paper proposed a decentralized control law for multi-vehicle formation. The proposed method has an information propagation structure between adjacent agents, so that the transient divergent behavior due to the partially informed situation was completely suppressed. The proposed method was found to have a drawback that the transient behavior of the formation can diverge when there is large information propagation delay, however, the divergent property can be controlled by appropriately selecting the feedback control gains.

This method was applied to one-dimensional car train control problem and two-dimensional satellite formation control problem, and the effectiveness was verified by numerical simulations.

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